

## CONTOUR SUB-DIVISION ALGORITHM FOR GRADIENT REFERENCE APPROACH TO MODEL HETEROGENEOUS OBJECTS

Vikas Gupta\*, MED, CDLMGEC Panniwala Mota, 9416434412, [vikasbrcm@rediffmail.com](mailto:vikasbrcm@rediffmail.com)

Rajesh Kumar Bansal, MED OITM Hisar, [rajeshbansal73@gmail.com](mailto:rajeshbansal73@gmail.com)

Vineet Kumar Goel, MED, OITM Hisar, [vineet13ss@gmail.com](mailto:vineet13ss@gmail.com)

**Abstract** The heterogeneous object consists of two or more materials with different characteristics with or without any specific boundary interface. The properties of the materials are controlled to functional requirement of the object. Rapid Prototyping processes have potential to fabricate such objects but require a CAD model with supplementary material information. The developed Gradient Reference approach has the capability to model the material distributions at different locations and assures the local control in the heterogeneous region. The approach presents a simple and efficient method to model the material distribution in heterogeneous objects. This paper focuses on exploring the contour sub-division algorithm to distribute the material in the object domain. The algorithm is capable of distributing the material in regular and irregular shape objects. The few examples are presented for validation of work. The developed approach is flexible and versatile to control the material composition at a location in HO. The material properties can also be evaluated for different material composition across the heterogeneous object.

**Keywords:** Heterogeneous Objects, gradient reference, contour sub-division algorithm

### 1. Introduction

Traditional CAD systems, used for conventional design method, can only represent the geometry and topology of an object. No material information is available within the representation which is required for heterogeneous objects. With the capability to fabricate heterogeneous objects, functionally efficient and cost reducing designs can be realized. Rapid prototyping (RP) techniques allow heterogeneous material objects to be produced using 3D CAD models by varying material composition region-wise, layer-wise, or point-wise. The required 3D CAD model should have not only the geometric information but also the information of material, property, etc. at each point inside an object. In order to take full advantage of the greatest potential of heterogeneous objects, one must have matching capabilities for their computer modeling, analysis, design optimization and visualization. The primary focus of the recent research development in these fields is on the computer representation schemes for heterogeneous objects, by extending the mathematical models and computer data structures of the modern solid modeling techniques to include discrete material regions of interfacial boundaries and heterogeneous properties. Recent studies show that an effective heterogeneous CAD modeling system should at least meet the various specifications [8]. Approaches of modeling of HO have been extensively studied in computer and manufacturing community. Kumar and Dutta [9-10] proposed an approach to model multi-material objects based on R-m sets and R-m classes primarily for application in layered manufacturing. Boolean operators were defined to facilitate the modeling process. Jackson [7] and Liu [11-12] proposed a local composition control (LCC) approach to represent heterogeneous object in which a mesh model is divided into tetrahedrons and different material compositions are evaluated on the nodes of the tetrahedrons by using Bernstein polynomials. Chiu [2] developed material tree structure to store different compositions of an object. The material tree was then added to a data file to construct a

modified format being suitable for RP manufacturing. Siu and Tan [16] developed a scheme named 'source-based' method to distribute material primitives, which can vary any material with an object. The feature-based modeling scheme was extended to heterogeneous object representation through boundary conditions of a virtual diffusion problem in the solid, and then designers could use it to control the material distribution and described by Qian and Dutta [14-15]. Liu extended his work in by taking parameterized functions in terms of distance(s) and functions using Laplace equation to smoothly blend various boundary conditions, through which designers could edit geometry and composition simultaneously [11-12]. Kou and Tan [8] suggested a hierarchical representation for heterogeneous object modeling by using B-rep to represent geometry and a heterogeneous feature tree to express the material distributions. Various methods for designing and optimizing objects composed of multiple regions with continuously varying material properties have been developed. Biswas and Wang [1],[16] proposed a level-set based variational scheme. Tsukunov [17] presented a mesh-free approach based on the generalized Taylor series expansion of a distance field to model and analyze a heterogeneous object satisfying the prescribed material conditions on a finite collection of material features and global constraints. However, almost all of the research interests are mainly focused on the computer representation of heterogeneous object, rather than the procedure for rapid prototyping and fabrication of heterogeneous object. The approaches were verified in commercial software packages, such as Solidworks and Unigraphics [11],[14]. A commercial CAD package independent system is developed to deal with the HO modeling, but not including the slicing procedure for RP manufacturing [13],[15]. In this paper, we just address the CAD gradient reference model with systematic methodologies for representation, visualization and manipulation of heterogeneous objects. A detailed description of each module cannot be presented for the paper length.

This paper is organized as follows: Section 2 represents gradient reference based CAD model with various material information functions and address local control in graded regions; contour sub-division algorithm is elaborated in Section 3; the few examples are presented in Section 4 and Section 5 concludes the paper.

## **2. Gradient Reference approach for computer aided modeling of HO**

The gradient reference approach reduces various limitations, hence, is more comprehensive to create heterogeneous objects as contrast to already presented methods [3-6]. The gradient references approach simply requires addressing following aspects for modeling of any HO.

- First, the Geometric Space  $G^3$  of an object is identified for the volume definition of heterogeneous object space also known as gradient space. The gradient space is a closed volume which is defined by a set of boundaries known as Boundary Enclosures (BE). Boundary closures can be defined by surfaces of the heterogeneous object and/or user defined surfaces, termed as Real (RS) and Virtual Surfaces (VS), respectively.
- Secondly, mode of material distribution is defined for a required material gradient in a gradient space. Mode of material distribution is simply the means by which material

- is distributed in the gradient space and is defined by introducing Gradient References (GR). The gradient references are the basic entities i.e. point, line/curve and plane/surface, in or out of geometric space, which originates the gradient space and limit it to the defined boundary enclosures in the object. Some features of the object i.e. plane/surfaces can be recognized commonly as gradient reference and boundary enclosures.
- Third feature to be defined is the material composition at the ends of gradient space in the direction of material variation. Distance is used as a parameter to define the composition of a point in the gradient space. Distance based material distributed functions are defined to vary the material composition across the gradient space. A mathematical model representing material distribution gradient function and related work is elaborated in previous work.

The above aspects of gradient reference approach are illustrated in Figure 1. Gradient space is represented by six boundary enclosures i.e. two virtual surfaces ( $VS_1$  and  $VS_2$ ) and four real surfaces ( $RS_1$ ,  $RS_2$ ,  $RS_3$  and  $RS_4$ ). Gradient direction along with gradient references ( $VS_1$  and  $VS_2$  are also act as GR) is defined. Material composition at start and end of the gradient region is defined by  $M_{c(s)}$  and  $M_{c(e)}$  respectively.

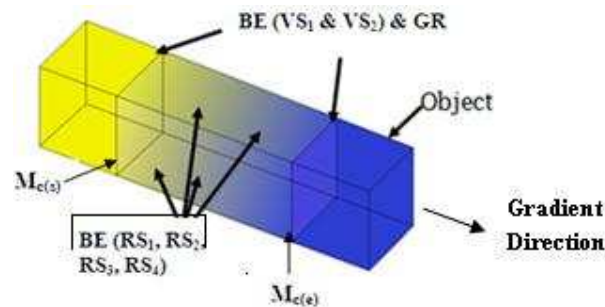


Figure 1: Aspects of Gradient References Approach.

By controlling and manipulating the gradient references and boundary enclosures, complex shape heterogeneous objects can be realized. In this approach, the basic entities used in defining the geometry of the object can be used as gradient references/boundary enclosures. However, user can create own references independent of geometry of the object for local control in HO.

### 3. Contour sub-division concept for GRA

As discussed above, the gradient space in the heterogeneous object domain is bounded by boundary enclosures. The pattern of material distribution may be associated with boundary enclosures thus depends upon the geometry of the object. In such cases, the gradient references and boundary enclosures may be of similar or dissimilar shape. Dissimilar gradient references/boundary enclosures are defined by two different shape entities in the model i.e. a square plate with a circular hole. Material distribution function for such object will adapt the shape of boundary enclosures. Material distribution for similar/dissimilar shape GR/BE is performed using a contour sub-division algorithm.

### 3.1 Algorithm

In order to distribute the material in HO, 2D solid area of 3D object, containing GR, BE and gradient direction information is considered. End contours (i.e. GR and BE) in the direction of gradient are identified. The gradient region between end-contours is divided into number of sub-regions. Each region has a constant material composition, but varies region-wise in the gradient direction. Contour sub-division algorithm is used to divide the gradient region into sub-regions through generation of q number of contours (including end contours).

The basic steps of algorithm are discussed below:

- (a) First p number of lines having  $l_p$  at equal interval (linear or angular) are drawn in 2D solid area between end contours.
- (b) Each line is divided in to q-1 number of small line segments. The size of  $i^{\text{th}}$  small segment ( $dl_{ip}$ ) of  $p^{\text{th}}$  line is achieved by dividing length of line ( $l_p$ ) by q-1.

$$dl_{ip} = \frac{l_p}{q-1} \dots \dots \dots (1)$$

$$\text{and } l_p = \sum_{i=1}^{q-1} dl_{ip} \dots \dots \dots (2)$$

- (c) The corresponding points on each line segment are joined to form q number of contours i.e.  $C_1, C_2 \dots C_q$  (including end contours).
- (d) 2D solid sub-regions ( $R_1, R_2 \dots R_{q-1}$ ) are created from adjacent contours. The accuracy of material distribution depends upon number of sub-regions, hence on number of contours.
- (e) To evaluate the properties of material composition at a point in a 2D solid area, it is necessary to incorporate primary material information and material distribution functions into the contour sub-division algorithm. Material composition at end contours is mapped into RGB colors, as represented in Eqn.3. Finally, a distance based function  $f(x)$  with RGB color coding is implemented for defining color (material) variation in a 2D solid area in sub-regions, see Eqn. 4.

$$M_{c(s)} \leftrightarrow (RGB)_s \text{ and } M_{c(e)} \leftrightarrow (RGB)_e \dots \dots \dots (3)$$

$$(RGB)_{q-1} = \nabla f(x) \times ((RGB)_e - (RGB)_s) + (RGB)_s, \text{ where } 0 \leq \nabla f(x) \leq 1 \dots \dots \dots (4)$$

## 4. Example

### 4.1 Example-1

Various steps of algorithm are illustrated in Figure 2, in which two non-concentric closed entities are end contours and the material distribution is in radial direction.

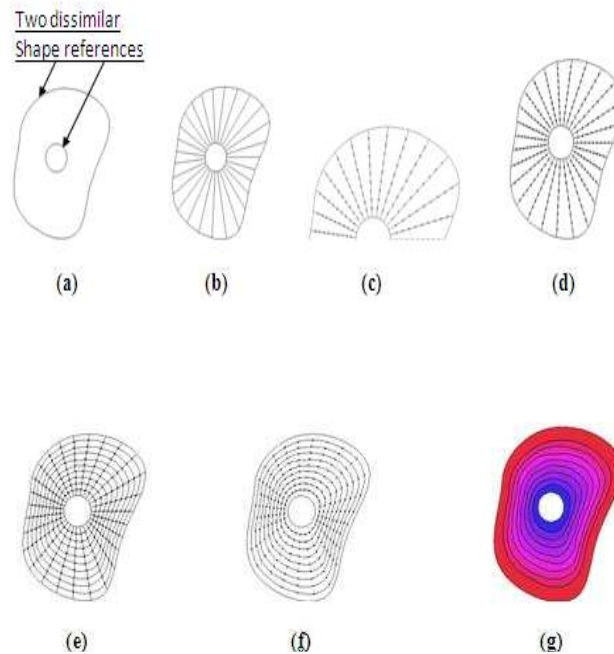


Figure 2: Contour sub-division algorithm: (a) selecting GR/BE, (b) drawing p number of lines, (c) division of each line into q-1 number of equal line segments, (d) full view of sub-divided object, (e) joining all respective points to get q number of contours, (f) removal of division lines and creation of q-1 number of sub-regions, and (g) material distribution using RGB color mapping.

#### 4.2 Example-2

Another example is illustrated in Figure 3 to model the material distribution in linear gradient direction between non-parallel open entities.

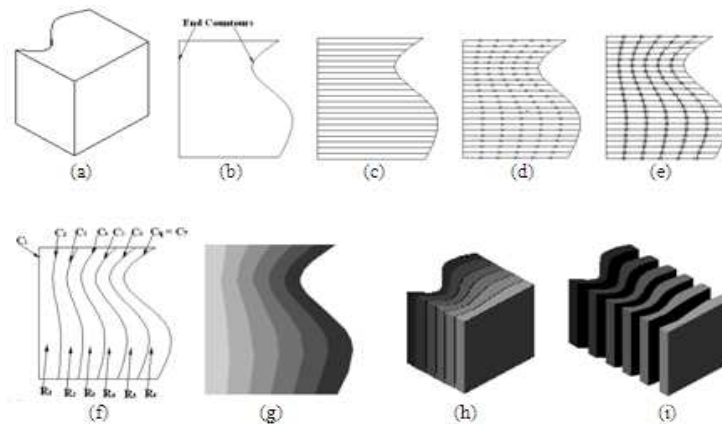


Figure 3: Contour sub-division algorithm for material distribution in a HO: (a) 3D geometry of object, (b) end contours/gradient references, (c) straight line division of 2D solid area, (d) dividing each line in equal

same size line segments, (e) contour generation, (f) generation of sub-regions, (g) material distribution in each sub-region, (h) material distribution extended to 3D object space, and (i) 3D homogeneous sub-spaces

## 5. Conclusion

This work presents contour sub-division algorithm for modeling material variations in simple and complex shape heterogeneous object. The developed gradient reference approach represents intricate geometries as well as material variation simultaneously; ensures smooth material variations throughout the object; imposes independent material alterations on the cells so that their original properties can be properly retained in the resultant object; offers local control on material distribution; consistent in data representation; and computationally robust and efficient.

The present work can be further extended and implemented complex and irregular material distributions. The approach can be extended to object modeling i.e. solid modeling with other physical attributes such as mechanical properties, material distribution etc. Dynamic heterogeneous objects (DHO) are the new class of heterogeneous objects. Unlike

## 6. References

- [1] Biswas, A.; Shapiro, V.; Tsukanov, I.: Heterogeneous material modeling with distance fields, *Computer-Aided Geometric Design*, 21(3), 2004, 215–42.
- [2] Chiu, W.-K.; Tan, S.-T.: Multiple material objects: from CAD representation to data format for rapid prototyping, *Computer-Aided Design*, 32, 2000, 707–17.
- [3] Gupta V, Bajpai VK, Tandon P. Slice generation and data retrieval algorithm for rapid manufacturing of heterogeneous objects. *Computer Aided Design & Applications* 2014;11(3):255-262. DOI: 10.1080/16864360.2014.863483
- [4] Gupta V, Kasana KS, Tandon P. Reference based geometric modeling for heterogeneous objects. *Computer Aided Design & Applications* 2012; 9(2): 155-165.
- [5] Gupta V, Kasana KS, Tandon P. CAD modeling, algorithm design, and system structure for rapid prototyping of heterogeneous objects. *International Journal of Computer Applications in Engineering, Technology and Sciences* 2010; 2(2): 299-303.
- [6] Gupta V, Kasana KS, Tandon P. Computer aided design modeling for heterogeneous objects, *International Journal of Computer Science Issues* 2010; 7(5): 31-38.
- [7] Jackson, T. -R.: Analysis of functionally graded material object representation methods, PhD thesis, Massachusetts Institute of Technology (MIT), Cambridge, MA, January 2000.
- [8] Kou, X.-Y.; Tan, S.-T.: A hierarchical representation for heterogeneous object modeling, *Computer-Aided Design*, 2004.
- [9] Kumar, V.; Dutta, D.: An approach to modeling and representation of heterogeneous objects, *Journal of Mechanical Design*, 120, 1998, 659–67.

- [10] Kumar, V.; Dutta, D.: An approach to modeling multi-material objects, Proceedings of the Fourth ACM Symposium on Solid Modeling and Applications, 1997, 336–45.
- [11] Liu, H.: Algorithms for design and interrogation of functionally graded material solids, Master's thesis, Massachusetts Institute of Technology, Cambridge, MA, January 2000.
- [12] Liu, H.; Maekawa, T.; Patrikalakis, N.-M.; Sachs, E.-M.; Cho, W.: Methods for feature-based design of heterogeneous solids, *Computer-Aided Design*, 36, 2004, 1141–59.
- [13] Marsan, A.; Dutta, D: On the application of tensor product solids in heterogeneous solid modeling, Proceedings of 1998 ASME Design Engineering Conferences, Atlanta, Georgia, September 1998, 1–9.
- [14] Qian, X.; Dutta, D.: Feature-based design for heterogeneous objects, *Computer-Aided Design*, 36(12), 2004, 1263–78.
- [15] Qian, X.; Dutta, D.: Physics-based modeling for heterogeneous object, *Trans ASME*, 125, September 2003.
- [16] Siu, Y.-K.; Tan, S.-T.: Source-based heterogeneous solid modeling, *Computer-Aided Design*, 34(1), 2002, 41–55.
- [17] Tsukanov, I.; Shapiro, V.: Meshfree modeling and analysis of physical fields in heterogeneous media, *Adv Comput Math*; 2003.
- [18] Wang, M.-Y.; Wang, X.-M.: A level-set based variational method for design and optimization of heterogeneous objects, *Computer-Aided Design*, 2004.